Virtual Radiometer Model

A Reverse Monte Carlo Ray Tracing approach to compute radiative fluxes to surfaces with user-specified view angles and orientations

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Motivation

• General need to compare radiative fluxes from experimental radiometers with fluxes computed in Thermal/Fluid simulations

• Current simulation suites lack ability to predict fluxes to objects with small view angles

• Validation efforts have suffered
Motivation cont’d

• DOM artifact: the ray effect
Narrow View Angle Gauge

- Ray effect exacerbated by narrow view angle gauges
Requirements

• Virtual Radiometer model
  – User-specified view angle
  – User-specified orientation
  – User-specified location
  – Fine angular discretization of rays within radiometer field of view
  – Fast
  – Accurate
Our Solution

• Reverse Monte Carlo Ray Tracing w/ following
  – Rays uniformly distributed across arbitrary solid angle
  – Rays rotated into an arbitrary direction
  – Refine only the rays that terminate on the radiometer; no need to refine entire domain
  – Mersenne Twister Random number generator
    • Non-repeating sequences > $2^{19,000}$
    • Fast, equidistributed random numbers
  – Angular discretization refined to arbitrary precision
Perhaps the most novel feature of this algorithm is the capability to handle rays used in the simulation. The radiative flux can then be calculated from the steradians, where each ray will subtend a part of a radiometer. Assuming uniform distribution of the rays, each ray can be weighted according to the solid angle that each part of a radiometer contributes. See Sun wu9q for the full derivation. The intensities from each of the rays can be obtained as:

\[ I_{i,k} = \int_{l_0}^{l_k} I_{b,cv}[T(l')] \kappa(l') \exp\left[-\int_{l'}^{l_k} \kappa(l'') dl''\right] dl' + I_{o,\text{sur}}(T_w) \exp\left[-\int_{l_w}^{l_k} \kappa(l') dl'\right] \]

After O(1,000) rays:

\[ q_i = \frac{\Omega}{N} \sum_{r=1}^{N} I_i(r) \cos(\theta(r)) \]
Challenges with Unstructured Meshes

Simple geometric relations make marching through the domain trivial

Geometric relations are much more complex in unstructured meshes
Dealing With Unstructured Meshes

• Two choices
  – Walk the node connectivity by calculating intersections
  – Select points along the ray, and query the elements to find which element owns those coordinates
Meshless Marching
Arbitrary View Angle

- The view angle defines a solid angle
- The rays must be distributed evenly throughout that solid angle
- Care must be taken when assigning values to traverse non-linear surfaces
- E.g. $4\pi$ sr. (a sphere)

Naïve approach

$$\phi_r = 2\pi R_1$$

$$\theta_r = \pi R_2$$

incorrectly distributed points

correctly distributed points
Correctly Distributed Points

\[
\phi_r = 2\pi R_1
\]

\[
\theta_r = \arccos (\cos(\theta_v) + \text{range} \times R_2)
\]

\[
\text{range} = 1 - \cos(\theta_v).
\]
All Possible View Angles
Arbitrary Orientation

- Unfortunately, radiometers in experimental setups are not always oriented along the same direction.
- The model must rotate the rays.

\[
A = \begin{bmatrix}
\cos \theta \cos \psi & -\cos \phi \sin \psi + \sin \phi \sin \theta \cos \psi & \sin \phi \sin \psi + \cos \phi \sin \theta \cos \psi \\
\cos \theta \sin \psi & \cos \phi \cos \psi + \sin \phi \sin \theta \sin \psi & -\sin \phi \cos \psi + \cos \phi \sin \theta \sin \psi \\
-sin\theta & \sin \phi \cos \theta & \cos \phi \cos \theta
\end{bmatrix}
\]

\[
\theta = \arccos \left( \frac{n_z}{\sqrt{n_x^2 + n_y^2 + n_z^2}} \right)
\]

\[
\psi = \arccos \left( \frac{n_x}{\sqrt{n_x^2 + n_y^2}} \right)
\]

\[
\text{if } (n_x, n_y) \in Q_3 : \psi = \frac{\pi}{2} + \psi_{\text{calculated}}
\]

\[
\text{if } (n_x, n_y) \in Q_4 : \psi = 2\pi - \psi_{\text{calculated}}
\]
All possible Orientations

\[ [A] x = b, \]

\[ \theta = \text{acos} \left( \frac{n_z}{n^2 x + n^2 y + n^2 z} \right) \]

\[ \psi = \text{acos} \left( \frac{n_x}{n^2 x + n^2 y + n^2 z} \right), \]

where \( n_x, n_y, \) and \( n_z \) represent the components of the vector normal of the radiometery. Note that there will never be a need to calculate a rotation about the \( x \) axis. All possible rotations can be accomplished using the other two while fixing \( \phi \) at 0. Due to the constraints of \( \text{arccos} \), with values of \( \psi \) must be adjusted if \( n_x, n_y \) are in the 3rd or 4th quadrants of the \( xy \) plane. Specifically, if \( n_x, n_y \) \( \in \mathbb{Q}_3 \):

\[ \psi = \frac{\pi}{2} + \psi \text{calculated} \]

if \( n_x, n_y \) \( \in \mathbb{Q}_4 \):

\[ \psi = 2\pi - \psi \text{calculated} \]

At this point, the rotation angles can be applied to the direction vectors of the rays, \[ [A] x = b, \]

where \( x \) is the pre-rotated direction vector of a ray in Cartesian coordinates, and \( b \) is the resulting direction vector.

3 Verification and Validation
3.1 Verification of Ray Distribution
To verify that given an orientation, a integer number of rays, and a view angle for a radiometer, that the appropriate equation 3 was implemented correctly, the model was used to solve for the view factor between a circular disc and an infinitesimal surface.

For a disc of radius \( r \) separated from the infinitesimal area by a distance \( h \), an analytical solution for the view factor is

\[ F = \frac{1}{(h/r)^2 + 1}. \]

The L2 error norm, defined as

\[ \sqrt{(A - C)^2} \]

where \( A \) is the exact solution and \( C \) is the computed solution, was shown to decrease with an increase in the number of rays such that at 100,000 rays, the L2 error norm was approximately 0.001. Because the distribution of the rays is of importance particularly for...
Needed

- Virtual Radiometer model
  - User-specified view angle
  - User-specified orientation
  - User-specified location
  - Fine angular discretization of rays that reach the radiometer
    - Fast
    - Accurate
Efficiency Considerations

• Random number generator: Mersenne Twister
• Avoid division: sigmaDivPi=sigma/pi;
• Take constants out of summation loops

\[ q_i = \frac{\sum_{r=1}^{N} I_i(r) \cos(\theta(r)) \frac{\Omega}{N}}{N} \]

• Storing parametric coordinates and the master element
  – If a set of rays is good enough for the first time step, why not re-use it?
  – Referencing an array much faster than element search for every step of every ray

\[ q_i = \frac{\Omega}{N} \sum_{r=1}^{N} I_i(r) \cos(\theta(r)) \]
Accuracy Considerations

- User controls # of rays
- User controls step size
- Incorporates dominant physics
  - Emitting/Absorbing media
  - Emitting walls
  - Reflective walls
Accuracy Considerations: Ray Convergence

\[ \sigma^2 \propto \frac{1}{N} \]
\[ \epsilon \propto N^{-\frac{1}{2}} \]
\[ \log(\epsilon) \propto \log(N^{-\frac{1}{2}}) \]
\[ \log(\epsilon) \propto -\frac{1}{2} \log(N) \]

\[ y = -0.5173x + 0.5798 \]
\[ R^2 = 0.98156 \]
Accuracy Considerations: Verification

• Compute view factor
• Compare to Analytical Solution

\[
F = \frac{1}{(h/r)^2 + 1}
\]

• Error < 0.2% with 100k rays
• Ray error error can be used a metric to determine confidence in ray distribution
Accuracy Considerations: Participating Media Verification

- Flux Divergence (W/m³)
  - Distance from center of domain (m)
  - Virtual Radiometer
  - Quasi-Exact Solution

10,000 rays
L2 Norm: 0.00305
Accuracy Considerations: Validation

18 inch Propellant fire Fuego Simulation  
18 inch Propellant fire Experiment
Validation Results

Radiative Flux (kW/m\(^2\))

Radiometer height

Emiss=1
Emiss=0.8
Emiss=0.2
Emiss=0
Emiss=0 || 0.8
DOM
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